**Brief Description:**

January through March—participants will learn about the fundamentals of sound and explore the mathematical relationships between the size of an instrument and the frequencies it produces. During their investigation, participants will explore the Physics of Sound and Mathematics of Music, design experiments and build musical instruments. They will explore the science behind sound, discover the mathematical relationships that rule the production of sound and music, and will explore how different instruments, (chordophones, aero phones, and idiophones) produce the same frequency although they sound different.

At the end of the module, participants will be able to play music with their instrument following mathematical patterns (repetition, transposition, mirror and others).

Participants will build 3 musical instruments: a Pythagorean scale monochord; a four-key wind chime; an equal tempered scale tuned xylophone.
LESSON OVERVIEW

Lesson 1 Subject and Topic: Introduction to Sound

Grade Level(s): 6-8

Brief Description of Lesson: Participants will be introduced to the physics of sound and mathematics of music through role play, discussion and the use of the appropriate software (oscilloscope and spectrum analyzer). Through the description of the module, they will be presented to the three instruments they will build during the module: a monochord, a wind chime and a xylophone.

STAGE 1: IDENTIFY DESIRED RESULTS

- **Enduring Understanding(s):**
  - Music is a popular method of expression.
  - Music is a collection of sounds that impacts the hearing.
  - Sound is a physical phenomenon that impacts hearing.
  - Sound is vibration that travels through air molecules moving from one place to another carrying with it energy (sound is a wave).
  - Sound can be characterized by certain parameters, objective ones (Intensity, Frequency and Quality of the sound) and subjective (volume, pitch and tone).

- **Essential question(s):**
  - Why study music?
  - When listening to music, what sense is impacted?
  - What is sound?
  - How sound is sound related to music?
  - What are the characteristics of sound?

- **Standards**
  - Next Generation Science Standards
    - MS-PS4-2. Develop and use a model to describe that waves are reflected, absorbed, or transmitted through various materials.
  - Common Core State Standards
    - CCSS.Math.Content.6.SP.B.5b Summarize numerical data sets in relation to their context, such as by describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

- **CCSS Standard for Mathematical Practice:**
  - CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.

- **Key Content Knowledge and Skills**
  - Students will know:
    - Physical and mathematical concepts can be used to describe the patterns and phenomena of sound and music.
    - Sound is mechanical vibration that travels through a media (generally air) moving from one place to another carrying with it energy. Sound is a Wave.
    - These molecules are in the air, and therefore air is the medium for sound travel.
    - Sound travels from a source to your ear. The sound wave carries energy that makes the tympanic membrane vibrates in the ear, then, these vibrations move the cochlea and its inner hairs, that transform these movements in electrochemical impulses interpreted by the brain as sound.
Sound has three main characteristics that we can use to identify it. These characteristics can be quantitative (objective) or qualitative (subjective) such as:

- Intensity (measure in dB - Objective) or volume (louder/softer - Subjective),
- Frequency (measured in Hertz Hz - Objective) or pitch (higher/lower - Subjective)
- Quality of sound (Harmonic analysis - Objective) or tone (quality - Subjective).

When identifying and recording observations of sounds, a scientist needs to be objective. This means describing the actual difference between sounds, and not the subjective observations about how those sounds make the scientist feel.

To make these observations objectively, we need an appropriate tool- An oscilloscope will be used to capture the sound and generate an image of the sampled sound as well as a Spectrum Analyzer, that will be used to visualize the harmonic composition of the sampled sound (in the module we are using a program called Audio Tester)

The Intensity of the sound and the frequency are measured with the software program Audio Tester. The intensity of the sound refers as the energy needed to move certain amount of area. Frequency refers to the number of vibrations that an individual particle makes in a specific period of time (measured in cycles per seconds or Hertz). Frequency is displayed through counting the number of patterns generated on the graph in a certain interval of time.

- Students will be able to:
  - Demonstrate the relationship between sound and vibrations through discussion.
  - Measure intensity and frequency using an oscilloscope and interpret the results.
  - Identify sound characteristics such as intensity, frequency and harmonic composition with their own voices and other sounds.

- **Academic Language Knowledge and Skills**
  - Students will know: Graphic patterns generated by the oscilloscope quantify and display the frequency of sound.
  - Students will be able to: Analyze and interpret graphs of different sound intensities by explaining which graphs represent higher/lower frequencies to the group.

- **English Language Development Knowledge and Skills (for starting, emerging, and developing ELLs)**
  - Students will know:
    - Vocabulary list: sound, frequency, intensity, tone, pitch, volume, vibration, wave and pattern.
  - Students will be able to (define by audience, behavior and conditions):
    - Use these terms in discussions with their peers and instructor.

**STAGE 2: DETERMINE ACCEPTABLE EVIDENCE**

- Pre-requisite/Prior knowledge for Both Content and Language:
  - N/A
- Formative Assessment for Both Content and Language
  - Participants will be able to talk about what sound is and the varying characteristics of sound.
  - Participants will be able to measure and analyze characteristics of sound waves generated by various sources of sounds.
- Other Forms of Assessment
○ The instructor will instruct the participants on how to use the “audio tester” (computer software program with oscilloscope) to measure the characteristics of sound. The instructor then will walk around and observe that participants are working together as they measure the characteristics of sound.

● Assessment Criteria
○ Participants will be able to measure the frequency of various sources of sounds using the “audio tester” with 80% accuracy.

● Summative Assessment (N/A)

STAGE 3: PLAN LEARNING EXPERIENCES

● Time Required for Segments
  ○ Set/Hook 15 minutes
  ○ Teacher Input 30 minutes
  ○ Guided Practice 40 minutes
  ○ Closure 5 minutes

● Grouping Arrangements
  ○ Whole class
  ○ Small group(s)
  ○ Pairs
  ○ Cooperative groups
  ○ Individual

● Materials and Technology
  ○ Use the Physics of Sound and Mathematics of Music Lesson Plan Handbook as an instructional tool.
  ○ Materials: The instructor will need to collect materials and toolbox from the Columbia College Chicago at 623 S. Wabash, room 600N.
    ■ Meter sticks (1 per participant)
    ■ Tuning fork 261 Hz (Do) and 552 Hz (Do)
    ■ Pretest, IRB Consent Forms and Participant weekly Surveys
  ○ Technology: Instructors will need to get access at their site, to use computers and a projector.
    ■ Computer outfitted with a microphone, Microsoft Excel and “audio-tester” software.
    ■ A projector to show participants the Scientists for Tomorrow website, handouts and audio-tester presentation.

● Teacher's Preparation
  ○ Assemble materials, practice procedures beforehand, identify new vocabulary, organize workstations, etc.
  ○ The teacher will need to make copies of the Pre-test.
  ○ The teacher should arrive at the site 15-30 minutes prior to the start of class.
  ○ The instructor will need to write definitions and diagrams on boards if available.
  ○ Watch video tutorials and lesson plans on how to use the audio tester.
  ○ Prepare Audio-tester using a projector, to present to the class.
  ○ As participants arrive, teacher should take attendance so they can fully complete instructor log afterwards.

● SET/HOOK
  ○ The instructor will begin with everyone introducing themselves and the Scientists for Tomorrow program in general, (instructor will bring up the website and show participants past photos and videos).
    ■ A program where you (the participants) can learn how to build musical instruments (Winter session), learn about solar energy and building a hybrid cart (Fall session), and finally learn about the importance of plants and people (Spring session).
    ■ For each session, you and an adult that you know will have the opportunity to attend Family Science Day Events at a museum or a conservatory, and explore more about what you have been learning in your after school activities.
Next, the instructor will introduce the 10-week module in which participants will develop and build 3 different “non-conventional” musical instruments.

- One instrument will be built with strings – a monochord.
- The second music activity will be building 2 instruments with vibrating solids- an idiophone (a tuned wind chime) and a diatonic equal tempered xylophone.

The instructor will explain the importance of the pre-assessment for the continuation of the program:

- The instructor will pass out the *Physics of Sound and Mathematics of Music* pre-assessment (*do not call them a test*).
- Participants will be assessed on their prior knowledge of sound as a phenomenon that follows the laws of physics and how music is based in mathematical principles. They will also be conducting a survey about their attitude towards STEM.

The instructor will introduce the concepts of Science, Technology, Engineering and Mathematics (STEM) and careers pertaining to the field.

- Anticipated Time: 15 minutes
- Pre-requisite/prior knowledge: None
- Formative Assessment: Observation and participation

**DEMONSTRATION/LECTURE:**

In an open forum, while recording the responses on the board, the instructor will ask participants for their opinions regarding sound:

- Why do we study music?
  - People love music and music is a popular method of expression.
- When listening to music, which sense is impacted?
  - Hearing
- What do we hear?
  - Sound
- What is sound and where does it come from?
  - Allow participants to list items that produce sound on the board and write out their own definitions.
  - To enhance the understanding of where sound comes from, the instructor will ask students to scream at the top of their lungs for 10 seconds. Afterwards, ask participants how they feel (they should feel tired). They feel tired because producing sound requires energy. Sound is a form of energy.

Next the instructor will ask, “If you can hear my voice when I am talking, what must the sound be able to do?” Guide participants to understand that sound travels from source to ear. Post the question, “How does sound travel?” and allow participants to discuss.

The instructor will demonstrate through role play the concepts of sound travel by having the participants line up and pretend to be molecules in air (air is composed of 78% Nitrogen, 21% Oxygen and 1% “other” gases).

Have participants line up close together to pretend to be molecules (arm to arm). The person at the start of the line will be pushed (by the instructor who is the energy) who will push their neighbor, who will in turn push the next neighbor, and so on. This will continue until the end of the line, and the last person in line will make a sound when it reaches them.

As students are standing in line, the instructor will ask, “As molecules, are you moving when you carry the sound?” The Participants will think about this. If necessary have participants...
watch their feet and complete the sequence again. Participants will notice that their feet do not move but the top of their bodies do. This slight movement will be referred to as vibration.

- After the role play demonstration the instructor will ask participants,
  - “What would happen if there were no molecules (no medium) and a sound is generated?”
  - Sound will not travel as there is no medium to carry it on.
- The instructor will ask the participants to place their hands on their throat and ask to repeat a pre-determined phrase:
  - Participants will describe what they felt on their throats
  - Emphasize the sense of vibration
- Instructor will ask the participants, “Did you hear your voice? Did you feel vibrations?
  - Emphasize connection between sound and vibrations
- To demonstrate the relationship between sound and vibrations:
  - The instructor will dangle a ruler off the edge of the table and firmly tap the dangling end of the ruler so that sound and vibrations are created (figure 1a)

Moving air molecules reach tympanic membrane of the ear and vibrations are interpreted by the brain (figure 1b & c)

- Conclude that sound is a phenomenon-a type of energy that travels through a medium from a source of sound through our ears. The movement of sound through the medium carrying with it energy, is called a wave. The molecules within the medium are vibrating but no one is changing their position (the energy travels but the molecules remain in the same place).
  - In physics, a wave is a disturbance (an oscillation) that travels through space and time, carrying with it energy. Similar to throwing a stone into a pond and causing “ripples” in water surface. There are different kinds of waves, mechanical (sound), electromagnetic (light) and others.

- Anticipated Time: 30 minutes
- Pre-requisite/prior knowledge: Vibrations, sound
- Formative Assessment: Observation and participation
GUIDED STUDENT PRACTICE

- The instructor will ask, “Are all sounds equal?” Participants will discuss and realize that sounds are different. The instructor can then ask, “How can we identify different sounds?” Guide the participants to understand that all sounds have particular characteristics that differentiate one sound to the other. These characteristics help us to identify them. Ask students, “What are some characteristics of sounds?” Participants can discuss volume (subjective), which is related to energy and intensity (objective), which can be measured.

- The instructor can demonstrate the concept of measuring objectively a characteristic by asking a participant to measure the teacher’s height using the ruler. As the student tries to measure, the instructor will move away and around to make it very hard. Ask students, “How can we measure something that is moving?” (The answer is by stopping it and then measuring.) This is a problem we have when we want to measure sound. Sound is moving all the time. The instructor will then explain that we have a measuring tool called an oscilloscope to help us measure sound. It can sample the mechanical vibration (sound) and convert into an image that we can freeze at any time we want. When the image is frozen we can proceed to measure the intensity and the frequency.

- The instructor will demonstrate how to use the audio tester software program on the computer:
  - First, the instructor will show participants how to set up the audio tester program, and then show them how to sample and measure a sound wave.
  - Participants will open the audio tester. The audio tester has three functions:
    - Oscilloscope: To visualize and measure sound waves
    - Frequency Generator: Generates any sound
    - Spectrum Analyzer: Measures frequency
  - (Note: If you find that the microphone is not picking up sound, click on parameter and make sure it is set to 16 bit, 44100 hZ.)

- The instructor will explain that an Oscilloscope is a device we use to visualize signals. The microphone is used to convert sound into electricity. The oscilloscope will convert these electrical signals in an image so we can see it. On the Oscilloscope screen, the horizontal axis is time and the vertical axis is intensity. Therefore, we are measuring the differences in the sounds intensity over time. Try it!!!

- Figure 2: Screenshot of the oscilloscope of audio-tester software. The oscilloscope shows the graphical representation of a sound. The vertical axis represents the intensity (in volts, after sound has been converted into electricity by the microphone) and the horizontal axis represents time
Figure 3: Screenshots of audio-tester profiles of a low intensity sound and a high intensity sound

- By setting the audio tester to 20mV and 1ms, you will see a very nice picture. However, even if the settings are different, the sounds and concepts are still the same.
- Next, have a volunteer participant vocalize a quiet “AAAAAHHHH” sound (after someone presses the start button) and the sound will be recorded using the audio-tester. A profile similar to that (figure 4) will be generated.

Figure 4: Analyzing the audio-tester profile of a low-frequency sound. Here we can count four to five repeating patterns in the view.

- The instructor will ask participants to describe what they see in the picture:
  - Emphasize that the picture shows a pattern—a phenomenon that repeats itself.
  - Participants will count the number of patterns they can see in the screen (between four and five, in figure 4). Point out where the pattern begins and ends.

- The instructor will repeat the same procedure as before, but this time have participants produce a “louder sound” and sample the voice. (A profile similar to that in (figure 5) will be generated.

Figure 5: Analyzing the audio-tester profile of a high-frequency sound. Here we can count 14 to 15 repeating patterns in the view.

- Participants will describe the pattern they see in the audio-tester profile:
  - Participants will count the number of patterns their sound produced in the audio tester profile (between 14 and 15 in figure 5).
  - The number of patterns present in the profile in figure 5 is more than the number of patterns present in the profile in figure 4, and the waves are different sizes. Explain to participants...
that by looking at the two different sounds, they are seeing two different patterns on the same amount of time.

- This is one characteristic of sound is called “Frequency.” The frequency is the number of patterns that a sound has in one second. Each sound has its own frequency. The unit of frequency is measured in Hertz (Hz)

- Next, the instructor will introduce participants to two different tuning forks. The instructor will ask, “Do these have the same frequency?” Walk around and allow students to try them out and decide which has the higher or lower frequency. The instructor will ask, “What can be the reason they do not have the same frequency?” Guide students to discuss the difference in the size of the tuning forks and how that relates to sound it produces.

- Participants will repeat the experiment described previously with two tuning forks of different sizes. Have participants hypothesize what the patterns will look like and guess which has the higher frequency.

- Measure the sound of the first tuning fork and calculate the frequency. Remember, the graph is showing intensity over time. We can then identify the pattern (how the intensity changes until it repeats itself). When we count the amount of patterns that the sound produces in one second, we receive the name of Frequency.

- Frequency is measured in Hertz (cycles/second) Count the number of patterns over the time base you selected, and convert this ratio to number of patterns over 1 second. This number should be pretty close to the number on the tuning fork. Complete this same process with the second tuning fork and confirm/reject the students’ hypotheses.

- Guide participants to understand that the shorter the tuning fork, the higher the frequency.

- Finally, relate these concepts to that of an orchestra. Appendix 1 provides a picture of an orchestra layout. Discuss with students how the violins are in the front, followed by the cello and then the bass. This is done this way because the bigger the instrument, the lower the frequency. This concept can also relate to the shape of a piano. The low notes have lower frequencies and therefore have longer strings. The high notes have higher frequencies and therefore have shorter strings. The curved, angular shape of a grand piano accounts for these differences in string length.

- Anticipated Time: 40

**INDEPENDENT STUDENT PRACTICE**

- Participants should visit the Scientists for Tomorrow website www.scientistsfortomorrow.org, as well as to visit Pinterest, Vimeo and Facebook, they understand that they are in an important program.

**CLOSURE**

- Review of what was learned in the first lesson.

- Instructor will discuss how measuring the frequency of sound will allow us to understand the musical scale.

- Anticipated Time: 5 minutes
LESSON OVERVIEW
Lesson 2 Subject and Topic: Building the monochord (Part 1)
Grade Level(s): 6-8
Brief Description of Lesson: Participants will review concepts learned in previous lessons and begin to build a monochord.

STAGE 1: IDENTIFY DESIRED RESULTS
   o Enduring Understanding(s):
     o A monochord is a one-string instrument used to demonstrate the mathematical relationship between the length of the string and the frequency it produces.
     o Precision in measurement is important because it can affect the final product we are designing and building. Wrong measurements will lead to the instrument being “out of tune.”
   o Essential question(s):
     o What is a monochord?
     o Why is measurement important?
   o Standards
     o Next Generation Science Standards
       ■ MS-PS4-2. Develop and use a model to describe that waves are reflected, absorbed, or transmitted through various materials.
     o Common Core State Standards
       ■ CCSS.Math.Content.7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
   o CCSS Standard for Mathematical Practice:
     o CCSS.Math.Practice.MP6 Attend to precision.
   o Key Content Knowledge and Skills
     o Students will know:
       ■ Tuning tools are used to tighten a tuning pin so a thread becomes more taught.
       ■ Measurement must be precise before cutting and gluing.
       ■ Using saws and drills require attention and safety procedures
     o Students will be able to:
       ■ Measure and cut the wood pieces precisely
       ■ Place dowels at a convenient interval using measurement appropriately
       ■ Use a drill to make a hole onto the wood safely
   o Academic Language Knowledge and Skills
     o Students will know: Measurement must be precise or sound can be affected.
     o Students will be able to: Summarize their use of a ruler and how they have sawed and cut precisely.
   o English Language Development Knowledge and Skills (for starting, emerging, and developing ELLs)
     o Students will know:
       ■ Vocabulary list: tuning wrench, tuning pin, saw, drill and scratch awl
     o Students will be able to:
       ■ Use these terms in conversation with their peers.

STAGE 2: DETERMINE ACCEPTABLE EVIDENCE
   o Pre-requisite/Prior knowledge for both Content and Language:
Students will be able to work cooperatively in groups.
Students will be able to use a ruler.

B. Formative Assessment for Both Content and Language
Participants will be able to work together to build their own monochords

Other Forms of Assessment
The instructor will observe for participation.

Assessment Criteria
The participants will build the monochords with 100% accuracy.

C. Summative Assessment: N/A

STAGE 3: PLAN LEARNING EXPERIENCES

Time Required for Lesson Segments
- Set/Hook: 5 minutes
- Teacher Input: 15 minutes
- Guided Practice: 45 minutes
- Closure: 15 minutes

Grouping Arrangements
- Whole class
- Small group(s)
- Pairs
- Cooperative groups
- Individual

Materials and Technology
Use the Physics of Sound and Mathematics of Music Lesson Plan Handbook as an instructional tool.
Materials: The instructor will need to collect materials from the Columbia College building at 623 S. Wabash in room 600N.

- Wood 1.5” x ¾” (approximately 85 cm per participant)
- 48 in. x ¼ in. Hardwood Round (Trimming wood) (Two 2cm pieces per participant)
- Tuning wrench
- Scratch Awl
- Ruler
- Tuning pins (1 per participant)
- Nylon Fishing string
- 1 drill
- 2 saws and mounts
- 1 Hammer
- 1 Brass Hook (1 per participant)

Technology: Instructors will need to get access from the site to use computers and a projector.
- Computer with Microsoft Excel and “audio-tester” software and outfitted with a microphone.
- A projector to show participants Scientists for Tomorrow website and handout Appendix 2.

Teacher's Preparation
- Assemble materials, practice procedures beforehand, identify new vocabulary, organize workstations, etc.
- As participants arrive, teacher should take attendance so they can fully complete activity log afterwards.
- Set up the classroom by putting four chairs per table.
- Set up the work stations
  - Drilling station
  - Saw station
  - Measuring and gluing Station
  - Tuning Station

Set/Hook
- The teacher will begin by reviewing some of the concepts from the first lesson such as:
  - Physical and mathematical concepts used to describe the patterns and phenomena of sound and music- Intensity and Frequency.
  - Sound is vibration that travels through air molecules moving from one place to another.
  - The vibrations require mechanical energy to travel from one place to another.
  - Sound travels from a source to your ear. The molecules vibrate the tympanic membrane in the ear and are then interpreted by the brain.
- Anticipated Time: 5 minutes
- Pre-requisite/prior knowledge: Discussion from previous week
- Formative Assessment: Observation

DEMONSTRATION/LECTURE:
- In an open forum, instructor will ask the participants:
  - “Who has ever used these tools before?”
  - “What is the tuning wrench used for?”
  - The instructor will allow students time to reflect and discuss their use of the tools.
- The instructor will show the participants the tools.
  - Discuss safety procedures
  - Show the students how to use the tools
  - The instructor will demonstrate how to use a ruler the correct way.
  - Have the students practice using a ruler by measuring an 85cm wood
- Anticipated Time: 15 minutes
- Pre-requisite/prior knowledge: Basic ruler use
- Formative Assessment: Observation of participation

Plans for Guided Student Practice
- The instructor will start the discussion by asking students, “What is a monochord?” Give students time to discuss what they think it is. Eventually guide students to understand that it is a one-string instrument. Use this instrument to demonstrate the mathematical relationship between the frequency and the length of the string. At the end of building a monochord the instructor will discuss how to design and plot all the notes from the diatonic scale.
- Next, the instructor will provide a piece of wood approximately 85 cm long to each participant. The instructor will ask: How can we drill one hole at an angle? Allow students to think about this and explain that they have to drill vertically (using a 3/16 BIT) to make a small mark on the
wood and then move the drill into a 70 degree angle. This hole will be 3 cm from the end of the wood. The instructor will first demonstrate participants how to drill a hole at one end to insert the tuning pin. (Remember the hole needs to be done at an angle. Use GOGGLES and the bit needs to be 3/16”!) 
- After the hole is made, the tuning pin will be placed slightly, and the pin will be further screwed in with the tuning wrench. Be sure to not screw the pin in too far as we will need to put string through the tuning pin hole.
  - Using a tuning wrench, screw tuning pins into the holes (figure 7a)

- Next, the participants will screw a brass hook into the side of the wood opposite of the tuning pin. The scratch awl can assist in making a small indent to screw into if needed.
- The instructor will then have students cut trimming wood pieces equal to the width of the wood (approximately 1.5 cm.). Have the student’s measure by themselves. They will then glue one piece of the molding 2cm from the tuning pin and the other on the edge of the opposite end.
- Next, participants will tie fishing string to the hook. They will then thread the opposite end through the tuning pin (figure 7b) and tighten (clockwise) with the tuning wrench. The thread needs to be tense in order for the sound to be perceived. Have students discover the tension they need and cut the excess string. (Don’t tighten too much or the string will break!)

- Anticipated Time: 40 minutes
- Pre-requisite/prior knowledge: 
- Formative Assessment: Observation

Plans for Independent Student Practice [Homework or independent practice related to the lesson]
- Participants should visit the Scientists for Tomorrow website www.scientistsfortomorrow.org, as well as following Pinterest, Vimeo and Facebook, so they understand what the program is about.

- Closure (brief teacher or student-led review, with reference back to essential questions and enduring understandings)
  - Plans: Review of what was learned in class.
  - Clean the room
  - Put the tools and material back to its place
  - Anticipated Time: 15 minutes
LESSON OVERVIEW

Lesson 3 Subject and Topic: Finding the relationship between the length of the string and the frequency it produces.

Grade Level(s): 6-8

Brief Description of Lesson: Participants will review concepts from Lessons 1 and 2, and further investigate the relationship between instrument size and sound frequency.

STAGE 1: IDENTIFY DESIRED RESULTS

● Enduring Understanding(s):
  ○ An open string is the length of the string that vibrates between the two nodes of the instrument. This length can only generate one frequency.
  ○ In the western culture, we use a musical scale called Equal tempered Scale, based in the one developed by the Greek mathematician Pythagoras 2600 years ago.

● Essential question(s):
  ○ How many frequencies can an open string generate?
  ○ Which frequencies do we want to produce in order to generate a pleasing musical composition (musical scale)?

● Standards
  ○ Next Generation Science Standards
    ■ MS-PS4-2. Develop and use a model to describe that waves are reflected, absorbed, or transmitted through various materials.
  ○ Common Core State Standards
    ■ CCSS.Math.Content.6.SP.B.5b Summarize numerical data sets in relation to their context, such as by describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

● CCSS Standards) for Mathematical Practice:
  ○ CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.

● Key Content Knowledge and Skills
  ○ Students will know:
    ■ By using an oscilloscope, sound can be measured by focusing on the differences between high and low frequencies.
    ■ Instruments are categorized by the source that produces the vibrations to generate the sound
    ■ The four types of instruments in an orchestra are Chordophones, Aerophones, Idiophones and Membranophones.
    ■ Frequency and Size are inverse proportional. The larger the size the lower the frequency and vice versa. Within the string group, the size is in this order: Bass (largest)- Low frequency> Cello-> "Guitar"-> Viola-> Violin (smallest) High Frequency
    ■ When plucking an open string, it can only generate one frequency.
    ■ The frequencies we selected to make a pleasing sound on any musical instrument are different from culture to culture. In the western music we use the Chromatic Equal tempered Scale (12 semitones per octave).
    ■ Pythagoras discovered these combinations (called Pythagorean ratios) by exploring the ratio between two identical shorter strings. Then finding the lengths of the short string with a perfect consonance sound with the constant one.
    ■ He found that by shortening the string to half of its length, we produce perfect consonance with the constant one. This length from the open string to half of the string receives the name of interval (then called the octave).
    ■ Because only two notes were not enough (open string and half of the string), Pythagoras continued to explore how to divide the interval in more notes, eventually deciding on seven
notes. A possible reason for the selection of the number seven is because at this time humans were aware of the existence of only seven celestial bodies. He believed that the movement of these bodies in the firmament produces a music called the Music of the Spheres, which made the universe harmonious. These notes are called Do Re Mi Fa Sol La and Ti. Do is the octave note and has a perfect consonant with the first note. That is the reason that both notes have the same name but a different frequency.

- Using the Pythagorean ratios allows musicians to produce instruments that will generate the same frequencies, allowing them to play together.

- Students will be able to:
  - Measure intensity and frequency using an oscilloscope
  - Interpret and use their knowledge of Pythagoras’ scale to finish building a monochord.
  - Measuring Frequency using the Spectrum Analyzer

**Academic Language Knowledge and Skills**

- Students will know: The diatonic scale can be utilized on a monochord by measuring the distances between frets.
- Students will be able to: Interpret the diatonic scale and accurately represent it on their monochord.

**English Language Development Knowledge and Skills (for starting, emerging, and developing ELLs)**

- Students will know:
  - Vocabulary List: Frequency, Vibrations, Pitch, Tone, Node, Diatonic musical scale, harmonious, octave and consonance.
- Students will be able to (define by audience, behavior and conditions):
  - Use these terms in conversation with their peers.

**STAGE 2: DETERMINE ACCEPTABLE EVIDENCE**

- Pre-requisite/Prior knowledge for Both Content and Language:
  - The frequency of sound is measured using the audio tester. Frequency refers to the number of vibrations that an individual particle makes in a specific period of time (measured in seconds). Frequency is measured by counting the number of patterns generated on the screen and the time the screen represents.

- **Formative Assessment for Both Content and Language**
  - Participants will continue building a monochord following the Pythagorean scale using the Pythagorean ratios.

- **Other Forms of Assessment**
  - The instructor will observe for cooperation and participation.

- **Assessment Criteria**
  - Participants will measure and build the monochord with 95% accuracy.

- **Summative Assessment: N/A**

**STAGE 3: PLAN LEARNING EXPERIENCES**

- **Time Required for Lesson Segments**
  - Set/Hook: 15 minutes
  - Teacher Input: 30 minutes
  - Guided Practice: 40 minutes
  - Closure: 5 minutes

- **Grouping Arrangements**
  - Whole class
  - Small group(s)
  - Pairs
  - Cooperative groups
  - Individual

- **Materials and Technology [LIST ALL RESOURCES]**
  - Use the *Physics of Sound and Mathematics of Music Lesson Plan* Handbook as an instructional tool.
Materials: The instructor will need to collect materials from the Columbia College Chicago at 623 S. Wabash, room 600N.

- Meter Sticks
- Monochord and Wedge
- Digital picture of an orchestra Appendix 1

Technology: Instructors will need to get access from the site to use computers and a projector.

- Computer with Microsoft Excel and “audio-tester” software and outfitted with a microphone.
- Projector to show participants Scientists for Tomorrow website and handouts.

Teacher's Preparation

- Assemble materials, practice procedures beforehand, identify new vocabulary, organize workstations, etc.
- The instructor will need to have computers and microphone with the audio tester
- Set up the classroom by putting four chairs per table.
- As participants arrive, teacher should take attendance so they can fully complete instructor log afterwards.

Differentiated or Individualized Learning (i.e. non-reader, ELL-levels, gifted)

- N/A

Set/Hook

- The instructor will begin by reviewing concepts addressed in the previous lessons
- As a whole group, the instructor will review measuring sound with the oscilloscope and measuring differences between low and high frequencies.
- The instructor will then introduce the Spectrum analyzer, which directly measures frequency.
- Next, using a projector, the instructor will present a picture of an orchestra.
- Have the participants point out the different instruments in an orchestra (Appendix 1)
- The instructor will explain how musical instruments are categorized by the vibrations’ source (string, air, solid or membrane)
- The instructor will show that there are 4 types of instruments in an orchestra:
  - Chordophones (Strings)
  - Aerophones (Air)
  - Idiophones (Solids)
  - Membranophones (Membrane)
- The instructor will ask, “Which is the largest instrument with strings? The smallest?”
  - Bass (largest): Cello and a Guitar
  - Smallest: Viola and violin
- Anticipated Time: 15 minutes
- Pre-requisite/prior knowledge: Concepts developed in the first two lessons.
- Formative Assessment: Observation of participation

DEMONSTRATION/LECTURE:

- The instructor will ask participants a series of questions and begin discussing the answers to these questions:
  - Q: How many frequencies can an open string generate?
    - An open string can produce only one frequency, allowing only monotone songs to be generated. To generate more interesting songs, we will need more “frequencies” (a.k.a. musical notes). In the previous lesson, we learned that different string lengths produce different frequencies.
  - Q: Which frequencies do we want to produce in order to generate a pleasing musical composition (a.k.a. a musical scale)?
Each culture has defined its own preferences for sounds that are “pleasant.” We are part of the Western culture and the musical scale developed by Pythagoras has influenced us.

**Q: How did Pythagoras begin to characterize a musical scale?**
- **A:** Pythagoras took a tool with two strings with equal length and tension (producing the same frequency) and started to shorten one of them to find the places where the two sounds produced by the strings, sound pleasant (are in consonance). He found that when we reduce a string to half of its original length, we have a perfect consonance (today called an interval or octave – a ratio of 2:1). Now we have two notes: the open string and the half-string. But these two notes are not enough so Pythagoras explored how to divide the string into more notes.

**Q: How did Pythagoras decide on the number of notes to be named?**
- **A:** Based on the myth of the music of the spheres (where the celestial bodies move producing music that makes harmony in the universe), Pythagoras decided to divide the string into seven intervals following the seven celestial bodies known at the time, a set of seven musical notes known today as the Diatonic scale (Do Re Mi Fa Sol La Ti). Note that when the next “Do” arises, the scale is repeated one octave up.

- Anticipated Time: 45 minutes
- Pre-requisite/prior knowledge: General knowledge of frequency developed previously.
- Formative Assessment: Observation/Checklist

**Plans for Guided Student Practice**
- The instructor will ask participants, “Does anyone here play an instrument with strings? How do you play it?”
  - Note: You change position of fingers to change the lengths of the strings (figure 8a)

- The instructor will then ask: based on experiments conducted in Lesson 1 what do you expect the relationship between the string length and the frequency to be?
- Participants will then hypothesize that the longer the string, the lower the frequency (and the shorter the string, the higher the frequency)
- As a group, conduct an experiment to test the hypothesis regarding string length and frequency. Provide all participants with a copy of the data table and graph paper provided in Appendix 2
- Assign 3 participants to conduct the experiment:
  - 1 participant will play pre-made monochord
  - 1 participant will operate computer
  - 1 participant will record data on the board or on a large piece of paper
  - Remaining participants will collect data into data table provided

- The participant playing the monochord will put a microphone in a box and lay the monochord on top. According to the lengths provided on the chart, the participant will measure the string from the base of the monochord (opposite of the tuning pin) and use a piece of trimming wood to hold the length. The participant running the computer will press start on the audio-tester spectrum analyzer (settings should be set between 100 Hz and 5kHz) (figure 9) while the monochord
string is plucked. The frequency will be recorded and the process will be repeated for each length.

- After the data is collected, the instructor will then enter data into the Excel Spreadsheet (frequencies should be recorded for string lengths of the open string length, 80, 70, 60, 50, 45, Half of the open string, 30, and a quarter of the open string. All the measurements are in centimeters.)

- Have participants graph the data using the graph paper provided in Appendix 2. Also, graph using a scatter plot in Excel

  Note: The following is data collected (sample data are presented in figure 10)

<table>
<thead>
<tr>
<th>String Length ($L_n$) (cm)</th>
<th>Frequency ($f_n$) (Hz)</th>
<th>Constant $K = L_n f_n$ (cm*Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1 = 90$</td>
<td>$f_1 = 200$</td>
<td>18000</td>
</tr>
<tr>
<td>$L_2 = 80$</td>
<td>$f_2 = 219$</td>
<td>17520</td>
</tr>
<tr>
<td>$L_3 = 70$</td>
<td>$f_3 = 255$</td>
<td>17850</td>
</tr>
<tr>
<td>$L_4 = 60$</td>
<td>$f_4 = 300$</td>
<td>18000</td>
</tr>
<tr>
<td>$L_5 = 50$</td>
<td>$f_5 = 360$</td>
<td>18000</td>
</tr>
<tr>
<td>$L_6 = 45$</td>
<td>$f_6 = 396$</td>
<td>17820</td>
</tr>
<tr>
<td>$L_7 = 40$</td>
<td>$f_7 = 443$</td>
<td>17720</td>
</tr>
<tr>
<td>$L_8 = 30$</td>
<td>$f_8 = 588$</td>
<td>17640</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>$L_0 = 22.5$</td>
<td>$f_0 = 799$</td>
<td>17978</td>
</tr>
</tbody>
</table>

Fig 10b

- Graph illustrates an inverse proportional curve (as string length decreases, frequency increases)
- $\frac{1}{2}$ string length = 2(frequency)
- $L_1 f_1 = L_2 f_2 = \ldots = L_n f_n = K$
  - Note: $L_1$ means string length 1; $f_1$ means frequency 1, etc.
- Provide participants with calculators (or phone) and ask them to determine the length of string that would be required to produce an example of a frequency of 350 Hz:
  - $L_1 f_1 = L_2 f_2$
  - $L_2 = L_1 f_1 / f_2$
  - $L_2 = (90 \text{ cm})(200 \text{ Hz}) / 350 \text{ Hz}$
  - $L_2 = 51.4 \text{ cm}$
  - Then test calculations by plucking a string of 51.4 cm and measuring the frequency (should be approximately 350 Hz).
- Reinforce that we are able to calculate the length of the string to produce the frequency that we want by knowing the length and the frequency of the open string.
  - In this way, a luthier (an artisan instrument builder) can calculate where to put the frets on a guitar so the player can produce the desired sounds.
  - This is science and mathematics applied to the arts.
- Anticipated Time: 40 min
  - Pre-requisite/prior knowledge: Using the oscilloscope and spectrum analyzer
  - Formative Assessment: Cooperation with groups

**Plans for Independent Student Practice**
- Participants should visit the Scientists for Tomorrow website www.scientistsfortomorrow.org, as well as to visit Pinterest, Vimeo and Facebook, so they understand that they are in an important program.

**Closure**
- Plans: Review of what was learned in the lesson and explanation of how this will be applied to our monochords during the next session.
- Anticipated Time: 5 min
LESSON OVERVIEW
Lesson 4 Subject and Topic: Building the monochord (Part 2)
Grade Level(s): 6-8
Brief Description of Lesson: Participants will learn to play music using a monochord

STAGE 1: IDENTIFY DESIRED RESULTS

- **Enduring Understanding(s):**
  - Precision in measurement is important because it can affect the work of the product we are building.
  - Notes can be changed by increasing or decreasing the length of the string in specific pre-calculated lengths.

- **Essential question(s):**
  - Why is measurement important?
  - How can an instrument with one string play various notes?

- **Standards**
  - Next Generation Science Standards
    - MS-PS4-2. Develop and use a model to describe that waves are reflected, absorbed, or transmitted through various materials.
  - Common Core State Standards
    - CCSS.Math.Content.7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

- **CCSS Standard for Mathematical Practice:**
  - CCSS.Math.Practice.MP6 Attend to precision.

- **Key Content Knowledge and Skills**
  - Students will know:
    - By changing the length of the string, we can produce different sounds/notes. Each note has a different frequency and will require a specific length of the string.
    - Pythagorean ratios (see figure 12 and Appendix 3) can be used to determine where to place the frets in the monochord (round dowels) to reduce the length of the string and generate notes with specific frequencies.

<table>
<thead>
<tr>
<th>Musical note</th>
<th>Pythagorean Ratio</th>
<th>Length of the string Ln</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1/1</td>
<td>L1</td>
</tr>
<tr>
<td>D</td>
<td>8/9</td>
<td>L1 * 8/9</td>
</tr>
<tr>
<td>E</td>
<td>64/81</td>
<td>L1 * 64/81</td>
</tr>
<tr>
<td>F</td>
<td>3/4</td>
<td>L1 * 3/4</td>
</tr>
<tr>
<td>G</td>
<td>2/3</td>
<td>L1 * 2/3</td>
</tr>
<tr>
<td>A</td>
<td>16/27</td>
<td>L1 * 16/27</td>
</tr>
<tr>
<td>B</td>
<td>128/243</td>
<td>L1 * 128/243</td>
</tr>
<tr>
<td>C</td>
<td>1/2</td>
<td>L1 * 1/2</td>
</tr>
</tbody>
</table>

Fig 12 Using Pythagorean Ratios

- Students will be able to:
  - Work together to change the length of the string on their monochord to make different sounds
  - Use Pythagorean ratios to determine where to place the frets on their monochord (round dowels)
  - Use their monochord and knowledge of notes to play a song
● **Academic Language Knowledge and Skills**
  ○ Students will know: By changing the length of the string at different points, the frequency of the sound will change.
  ○ Students will be able to: Explain the relationship between frequency, length and sound in discussion with their peers.

● **English Language Development Knowledge and Skills (for starting, emerging, and developing ELLs)**
  ○ Students will know:
    - Vocabulary List: monochord, measurement, increasing, decreasing, frequency, note and Pythagorean ratio,
  ○ Students will be able to (define by audience, behavior and conditions):
    - Use these terms in conversation with their peers during discussion.

**STAGE 2: DETERMINE ACCEPTABLE EVIDENCE**

● **A. Pre-requisite/Prior knowledge for Both Content and Language:**
  ○ Pythagoras discovered these combinations of notes by shortening and lengthening the tension of the string and finding which two lengths were harmonious.
  ○ He found that by reducing a string to half of its length, it produces a perfect consonance. This length receives the name of Interval or an octave.
  ○ Because only two notes are not enough, Pythagoras continued to divide the string’s length, eventually deciding on seven notes because there were seven celestial bodies; which he believed made the universe harmonious. These notes are called Do Re Mi Fa Sol La and Ti.

● **B. Formative Assessment for Both Content and Language**
  ○ Student Product and/or Performance
    - Participants will be able to work together to build their own monochords
    - Participants will be able to accurately measure the distance between the round dowels.

● **Other Forms of Assessment**
  ○ Observation of students participation and cooperation

● **Assessment Criteria**
  ○ Participants will measure with 90% accuracy.

● **C. Summative Assessment (N/A)**

**STAGE 3: PLAN LEARNING EXPERIENCES**

● **Time Required for Lesson Segments**
  ○ Set/Hook: 15 minutes
  ○ Teacher Input: 30 minutes
  ○ Guided Practice: 40 minutes
  ○ Closure: 5 minutes

● **Grouping Arrangements**
  ○ Whole class
  ○ Small group(s)
  ○ Pairs
  ○ Cooperative groups
  ○ Individual

● **Materials and Technology**
  ○ Use the *Physics of Sound and Mathematics of Music Lesson Plan* Handbook as an instructional tool.
  ○ **Materials:** The instructor will need to collect materials from the Columbia College building at 623 S. Wabash in room 600N.
    - Monochords built during last lesson
    - Round dowels
    - Wood Glue
    - Plier Cutters
    - Calculators
Teacher’s Preparation
- Assemble materials, practice procedures beforehand, identify new vocabulary, organize work stations, etc.
- As participants arrive, teacher should take attendance so they can fully complete instructor log afterwards.
- Set up the classroom by putting four chairs per table.
- Set up the work stations
  - Measuring and gluing Station
  - Tuning Station

Set/Hook
- The instructor will activate prior knowledge by asking the participants, “How can you play music with the monochords?” Participants will discuss the research they did last week by shortening and lengthen the monochord string to change the frequency or sound. The instructor will guide students to understand that we want to use those concepts to find the lengths of string to produce harmonious notes on our own monochords.
- The instructor can ask, “When playing a guitar, how does a musician know where to place their fingers?” The participants should discuss this and talk about how guitars have frets so musicians know at what points to shorten or lengthen the string. This will help to produce a desired and specific frequency (note).
- Anticipated Time: 5 minutes
- Pre-requisite/prior knowledge: Frequency concepts developed previously
- Formative Assessment: Observation

DEMONSTRATION/LECTURE:
- In an open forum, the instructor will ask the participants: “How do we know where to place our frets to create a musical scale?” The participants discuss different scales that are used in music and how those notes are related to the string length. Therefore, each note has a different frequency.
- The instructor will ask the participants what are the 8 popular notes:
  - (Do, Re, Mi, Fa, Sol, La, Ti, Do)
  - The instructor can discuss the name of the known planets at the Pythagoras time in Latin, music of the spheres, and cosmic music of the universe. This is why we have 7 musical notes.
- The instructor will explain that because the notes have individual frequencies, we can calculate where the frets need to go by measuring their own string.
- Anticipated Time: 15 minutes
- Pre-requisite/prior knowledge: Musical harmony
- Formative Assessment: Observation

Plans for Guided Student Practice
- The instructor will ask, “How can we figure out what the length of the string should be for each fret?”
- The instructor will explain that the ratios that Pythagoras invented can be applied to any length of string.
- The instructor will pass out the Pythagorean ratios worksheet and have students measure the length of string on their monochords.
<table>
<thead>
<tr>
<th>Musical Note</th>
<th>Pythagorean Ratio</th>
<th>Length of the String $L_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$1/1$</td>
<td>$L_1$</td>
</tr>
<tr>
<td>D</td>
<td>$8/9$</td>
<td>$L_1 \times 8/9$</td>
</tr>
<tr>
<td>E</td>
<td>$64/81$</td>
<td>$L_1 \times 64/81$</td>
</tr>
<tr>
<td>F</td>
<td>$3/4$</td>
<td>$L_1 \times 3/4$</td>
</tr>
<tr>
<td>G</td>
<td>$2/3$</td>
<td>$L_1 \times 2/3$</td>
</tr>
<tr>
<td>A</td>
<td>$16/27$</td>
<td>$L_1 \times 16/27$</td>
</tr>
<tr>
<td>B</td>
<td>$128/243$</td>
<td>$L_1 \times 128/243$</td>
</tr>
<tr>
<td>C</td>
<td>$1/2$</td>
<td>$L_1 \times 1/2$</td>
</tr>
</tbody>
</table>

Figure 12, Pythagoras worksheet example

- Participants will then use the length of their own monochord to calculate the ratios for the placement of their frets (round dowel pieces).
- The instructor will go around the classroom to see if any of the participants need help to solve the chart. Students can work on this individually, in pairs or in groups.
- After the participants have completed their chart, they will go to the board to discuss their findings and write their answers while the instructor evaluates.
- Next, the instructor will ask the students, “What size measurement should we cut the dowels that will be our frets?” Allow students to discuss their ideas and come to the understanding that they should measure the width of the monochord.
- The participants will then measure the width of their monochord and cut 7 pieces of the round wood dowel according to their measurements.
- Participants will measure the length on their monochords and glue the round dowels in appropriate places based on their previous calculations.
- After the frets (round wood dowels) are placed and the glue is dry, play a simple tune so the participants can understand the placement of the notes.
- Anticipated Time: 45 minutes
- Pre-requisite/prior knowledge: Pythagorean Ratios
- Formative Assessment: Observation and Presentation

- **Plans for Independent Student Practice** [Homework or independent practice related to the lesson]
  - Participants should visit the *Scientists for Tomorrow* website www.scientistsfortomorrow.org, as well as to visit Pinterest, Vimeo and Facebook, so they understand what the program is about.

- **Closure**
  - Plans: Review of what was learned in class and encourage students to learn more tunes.
  - Put the tools and material back to its place and clean the room
  - Anticipated Time: 5 minutes
LESSON OVERVIEW
Lesson 5 Subject and Topic: Experimenting with Aero phones
Grade Level(s): 6-8
Brief Description of Lesson: Participants will explore aero phones and how the sound is generated. They will measure the frequencies of the sounds generated by, and relate them to the lengths of the aero phones.

STAGE 1: IDENTIFY DESIRED RESULTS

● Enduring Understanding(s):
  ○ Aero phones produce sound by vibrating columns of air.
  ○ By making the air vibrate into different pipe lengths, we can measure the frequency that each different length produces. (Longer pipes have lower frequency, short pipes have higher frequency.)

● Essential question(s):
  ○ What is an aero phone?
  ○ What is the relationship between the length of the pipe (air column) and the frequency it produces when the air vibrates?

● Standards
  ○ Next Generation Science Standards
    ■ MS-PS4-2. Develop and use a model to describe that waves are reflected, absorbed, or transmitted through various materials.
  ○ Common Core State Standards
    ■ CCSS.Math.Content.6.SP.B.5b Summarize numerical data sets in relation to their context, such as by describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

● CCSS Standards) for Mathematical Practice:
  ■ CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.

● Key Content Knowledge and Skills
  ○ Students will know:
    ■ If a pipe is cut in half, the frequency generated will be double the frequency of the original pipe. The half pipe is providing a note that is the same note that the full pipe has but one octave higher. A quarter of the original pipe will generate a two octave note higher from the original pipe.
    ■ The diameter of the pipe does not affect the frequency of the sound it produces.
    ■ Basic math concepts: inverse proportional and finding a constant
  ○ Students will be able to:
    ■ Find the constant K needed to calculate the inverse proportionl relationship from an experiment
    ■ Blow a pipe and measure the frequency it produces
    ■ Collect data from different lengths of pipes, make a graph of the collected data and discuss your interpretation of the data

● Academic Language Knowledge and Skills
  ○ Students will know: By changing the length of the pipe, the frequency of the sound will change.
  ○ Students will be able to: Explain the relationship between frequencies, length and sound while discussing with their peers.

● English Language Development Knowledge and Skills (for starting, emerging, and developing ELLs)
  ○ Students will know
    ■ Vocabulary list: aero phone, constant, frequency
Students will be able to (define by audience, behavior, conditions):

- Use these terms in conversation with their peers

STAGE 2: DETERMINE ACCEPTABLE EVIDENCE

- A. Pre-requisite/Prior knowledge for Both Content and Language:
  - Frequency concepts from previous lesson
  - Appropriate use of the Audio tester software program

- B. Formative Assessment for both Content and Language
  - Participants will be able to work with their peers to explore frequency differences.

- Student Product and/or Performance
  - Participants will be able to discuss their findings
  - Participants will be able to analyze and solve for the constant K

- Other Forms of Assessment
  - Observation of student participation

- Assessment Criteria
  - Students will measure frequencies with 90% accuracy

- Summative Assessment (N/A)

STAGE 3: PLAN LEARNING EXPERIENCES

- Time Required for Lesson Segments
  - Set/Hook: 15 minutes
  - Teacher Input: 30 minutes
  - Guided Practice: 40 minutes
  - Closure: 5 minutes

- Grouping Arrangements
  - Whole class
  - Small group(s)
  - Pairs
  - Cooperative groups
  - Individual

- Materials and Technology
  - Use the Physics of Sound and Mathematics of Music Lesson Plan Handbook as an instructional tool.
  - Materials: The instructor will need to collect materials from the Columbia College Chicago at 623 S. Wabash, room 600N.
    - Pipes of various lengths
    - Pipes of various (optional)
  - Technology: Instructors will need to get access from the site to use computers and a projector.
    - Computer with Microsoft Excel and “audio-tester” software and outfitted with a microphone.
    - A projector to show participants the Scientists for Tomorrow website and handouts.

- Teacher's Preparation
  - Assemble materials, practice procedures beforehand, identify new vocabulary, organize workstations, etc.
  - Have the computers ready with microphone and excel.
  - Have the projectors ready
  - As participants arrive, teacher should take attendance so they can fully complete instructor log afterwards.

- Set/Hook
  - The instructor will take one pipe and blow it to demonstrate what an aerophone is. Have participants think about the frequency and the length of the pipe. Lead the conversation by doing an experiment to find the relationship between the length and the frequency in the aero phone. (Experiment is similar to the string experiment)
  - The instructor will teach the participants how to cut a pipe using a pipe cutter. Explain to the participants that they need to carefully place the blade into the copper. They should not tighten the cutters to the copper to the maximum this will create dents to the pipe.
○ In groups of two the participants will cut pipes in the following lengths in cm 32.6, 29, 26, 24.5, 21.8, 19.5, 17.3, and 16.3 (these lengths are tuned to the notes: Do Re Mi Fa Sol La Ti Do)
○ The instructor will begin by performing a short tune on the “flute.” The instructor will explain that we call these instruments aerophones. They produce sound by vibrating air in the solid.
○ Once the pipes are cut, the instructor will align 8 participants from shortest to tallest. Give each pipe (corresponding to their height).
○ The instructor will demonstrate how to blow the pipe.
  ■ Place your thumb at the bottom of the pipe and put the pipe near your chin and blow
○ Point to any of the eight different participants and have them blow into the pipe to play a song as a group. Have the “audience” realize that the shortest and longest pipes sound nice together and are 1 octave apart.
○ Using the cut pipes, the instructor will lead an experiment to relate the frequency to the length of pipe using the table and graph in Appendix 4. Participants will collect the data and construct a graph that will show the relation between length and frequency.
  ■ Note that the instructor will need to learn how to blow pipes because it can be challenging for participants – it is trivial that the pipes sound good enough, to measure well!!!!
○ Instructor will use techniques used in lesson 2 to establish the inverse proportional relationship between the frequency generated and the length of the pipe (L_1f_1= L_2f_2=K[constant])
  ■ Participants will find that L_1f_1= L_2f_2= ...= L_nf_n=K and therefore f_n= K/L_n where K is a constant. The graph will show that the frequency is inversely proportional to the length of the air column and as in the string, if we cut the pipe in half, the frequency is the double. Cutting the pipe in half is producing the double of the frequency (same note one octave up)
○ Anticipated Time: 15 minutes
○ Pre-requisite/prior knowledge: Careers in STEM

**Plans for Guided Student Practice**

Find the frequency for each of the pipes by having someone blow into each of the different pipe lengths and then measuring with the audio tester to make a table.
○ The Spectrum Analyzer settings should be set to FFT Window: Hamming, FFT Size: 4096, Input: Left L_1 and Right L_2, Averages: 1, Mode: Continuous
○ The instructor can have one participant write the data on the board, another starting and stopping the audio tester, another blowing into the pipes and another typing numbers into an excel sheet.
○ Be sure that the microphone is set up correctly, such that it is not in front of the mouth of the person playing the aerophone. (This will change the frequency.)
○ The typed data can be highlighted and turned into a scatter plot
○ The data should look similar to that of Figure 12a
<table>
<thead>
<tr>
<th>Pipe Length ($L_n$)</th>
<th>Frequency ($f_n$)</th>
<th>Constant $K = L_n f_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(cm)</td>
<td>(Hz)</td>
<td>(cm*Hz)</td>
</tr>
<tr>
<td>L₁ = 32</td>
<td>$f_1 = 263$</td>
<td>8416</td>
</tr>
<tr>
<td>L₂ = 30</td>
<td>$f_2 = 276$</td>
<td>8280</td>
</tr>
<tr>
<td>L₃ = 28</td>
<td>$f_3 = 300$</td>
<td>8400</td>
</tr>
<tr>
<td>L₄ = 26</td>
<td>$f_4 = 325$</td>
<td>8450</td>
</tr>
<tr>
<td>L₅ = 24</td>
<td>$f_5 = 345$</td>
<td>8280</td>
</tr>
<tr>
<td>L₆ = 22</td>
<td>$f_6 = 374$</td>
<td>8228</td>
</tr>
<tr>
<td>L₇ = 20</td>
<td>$f_7 = 415$</td>
<td>8300</td>
</tr>
<tr>
<td>L₈ = 18</td>
<td>$f_8 = 460$</td>
<td>8280</td>
</tr>
<tr>
<td>L₉ = 16</td>
<td>$f_9 = 513$</td>
<td>8208</td>
</tr>
<tr>
<td>L₁₀ = 14</td>
<td>$f_{10} = 580$</td>
<td>8120</td>
</tr>
<tr>
<td>L₁₁ = 12</td>
<td>$f_{11} = 667$</td>
<td>8004</td>
</tr>
<tr>
<td>L₁₂ = 10</td>
<td>$f_{12} = 781$</td>
<td>7810</td>
</tr>
</tbody>
</table>

Fig 12b example of the data

- Anticipated Time: 40
- Pre-requisite/prior knowledge:
- Formative Assessment:

- **Plans for Independent Student Practice**
  - Participants should visit the *Scientists for Tomorrow* website www.scientistsfortomorrow.org, as well as to visit Pinterest, Vimeo and Facebook, so they understand that they are in an important program.

- **Closure**: Brief teacher or student-led review, with reference back to essential questions and enduring understandings
  - Plans: Review of what was learned in the previous lesson.
  - Instructor will discuss how measuring the frequency of sound will allow us to understand the musical scale.
  - Anticipated Time: 5 minutes
LESSON OVERVIEW
Lesson 6 Subject and Topic: Introduction to idiophones
Grade Level(s): 6-8
Brief Description of Lesson: Participants will be introduced to idiophones (vibrating solids), including the xylophone, marimba and triangle.

STAGE 1: IDENTIFY DESIRED RESULTS

- **Enduring Understanding(s):**
  - The frequency is inversely proportional to the squared length of the pipe.

- **Essential question(s):**
  - How does the length of an idiophone affect the frequency differently from that of a string or aerophone?

- **Standards**
  - Next Generation Science Standards
    - MS-PS4-2. Develop and use a model to describe that waves are reflected, absorbed, or transmitted through various materials.
  - Common Core State Standards
    - CCSS.Math.Content.7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

- **CCSS Standard for Mathematical Practice:**
  - CCSS.Math.Practice.MP6 Attend to precision.

- **Key Content Knowledge and Skills**
  - Students will know:
    - How to use tools in a safe manner
    - Difference from the string and aerophone concepts, the frequency of the idiophone is inversely proportional to the squared length of the pipe. If the pipe is cut in half, the frequency will rise four times.
    - How to use a pipe cutter
    - Learn how to measure and cut properly
  - Students will be able to:
    - Measure and Cut pipes
    - Calculate the length of a pipe to produce the desired frequency
    - Apply their knowledge of frequency by using the audio tester program

- **Academic Language Knowledge and Skills**
  - Students will know: Pipes must be measured precisely so they are harmonious according to the musical scale desired.
  - Students will be able to: Compare and contrast the differences in frequency and scales.

- **English Language Development Knowledge and Skills (for starting, emerging, and developing ELLs)**
  - Students will know:
    - Vocabulary list: idiophone, frequency and constant
  - Students will be able to:
    - Use these terms in conversation with their peers.

STAGE 2: DETERMINE ACCEPTABLE EVIDENCE

- Pre-requisite/Prior knowledge for Both Content and Language:
○ Use concepts learned in previous lessons to determine the relationship between the length of the vibrating solid and the frequency of the sound produced.
○ Pythagoras discovered these combinations by shortening and lengthening a tense string, and finding which two lengths were harmonious.
○ Pythagoras found that by reducing half of its length, we produce the perfect consonance or an octave.
○ Because only two notes were not enough, Pythagoras continued to divide the string length, eventually deciding on seven notes because there were seven celestial bodies, he believed made the universe harmonious. These notes are called Do Re Mi Fa Sol La and Ti.

- Formative Assessment for Both Content and Language
  ○ Student Product and/or Performance
    ■ Students will cut pieces of pipes according to the frequencies of a musical scale.
  ○ Other Forms of Assessment
    ■ Students will work cooperatively with partners.
  ○ Assessment Criteria
    ■ Students will measure and cut pipes with 95% accuracy
- C. Summative Assessment (N/A)

STAGE 3: PLAN LEARNING EXPERIENCES

- Time Required for Lesson Segments
  ○ Set/Hook: 15 minutes
  ○ Teacher Input: 15 minutes
  ○ Guided Practice: 45 minutes
  ○ Closure: 15 minutes

- Grouping Arrangements
  ○ Whole class
  ○ Small group(s)
  ○ Pairs
  ○ Cooperative groups
  ○ Individual

- Materials and Technology
  ○ Use the Physics of Sound and Mathematics of Music Lesson Plan Handbook as an instructional tool.
  ○ Materials: The instructor will need to collect materials from the Columbia College building at 623 S. Wabash in room 600N.
    ■ 1.5 m of 1/2” diameter copper pipe
    ■ Pipe cutters
    ■ Rulers
    ■ Markers
    ■ Calculators
  ○ Technology: Instructors will need to get access from the site to use computers and a projector.
    ■ Computer with Microsoft Excel and “audio-tester” software and outfitted with a microphone.
    ■ A projector to show participants Scientists for Tomorrow website and handouts.

- Teacher's Preparation
  ○ Assemble materials, practice procedures beforehand, identify new vocabulary, organize workstations, etc.
  ○ As participants arrive, teacher should take attendance so they can fully complete instructor log afterwards.
  ○ Set up the classroom by putting four chairs per table.
  ○ Set up the work stations
    ■ Drilling station
    ■ Saw station
    ■ Measuring and gluing Station
    ■ Tuning Station
**Set/Hook**
- The teacher will begin by activating prior knowledge and asking questions about previous frequency concepts.
- Anticipated Time: 15 minutes
- Pre-requisite/prior knowledge:
  - The longer the string, the lower the frequency.
  - As with string, if you decrease the length of an aerophone by half, you will go up an octave.
- Formative Assessment: The teacher will observe for students who can demonstrate understandings during discussions with the whole group and peers.

**DEMONSTRATION/LECTURE:**
- In an open forum, instructor will ask the participants “What type of material is this pipe made from?” Students will discuss different types of metal and eventually come to the conclusion that it is copper.
- Next the instructor will ask, “How can we cut the pipe?” Students can review how they cut pipes in previous lessons and the instructor can perform a demonstration.
- Discuss safety procedures
  - Note: Make sure the pipe is lay between the two round turning wheels and the sharp turning wheel
- The instructor will ask students how they previously made sounds with the pipe. (They blew into the pipes)
- The instructor can then ask, “What is another way we can use the pipe to make sound?” Have participants discuss ideas in pairs before returning to the whole group. The instructor should guide students to understand that we can tap the pipe with another solid.
- The instructor will discuss idiophones and different examples such as the triangle, xylophone and chime. The instructor will ask what they think will happen to the frequency of the vibrating solid (idiophone) if we decrease the length by half. Students will hypothesize that the frequency will be doubled, as this was the case for both strings and aero phones.
- Test the hypothesis for vibrating solids, \( \frac{1}{2} \) (length) = 2(frequency)
  - NOTE: data generated will not support this hypothesis.
  - Ask participants to design an experiment to test the proposed hypothesis using the techniques learned in previous lessons.
- Have participants list the materials and technologies that will be required to conduct the experiment:
  - Pipes of different lengths from the same original pipe to ensure the same density and shape
  - Spectrum Analyzer (audio-tester software)
- Anticipated Time: 45 minutes

**Plans for Guided Student Practice**
- Demonstrate to participants how to cut pipes
  - Measure 22 cm of pipe (Figure 13a)
  - Remind participants that an error of 1 mm makes a big difference in the results (measurement is critical!)
  - Align the pipe cutter blade with the marker line and secure the pipe cutter in place (Figure 13b)
  - Rotate the pipe cutter around the pipe creating a groove in the pipe (Figure 13c)
  - Tighten the pipe cutter around the pipe gently and rotate again and repeat until the pipe is cut (Figure 13d)
Assist participants to cut pipes of 22, 20, 18, 16, 14, 12, and 10 cm (Figure 13e)

- Ask participants to re-measure the lengths of the cut pipes and write the exact length on the pipe (Figure 13f)
- Set up a “pipe testing station” where the frequencies of the cut pipes will be tested. (Figure 14a & 14b)

- Collect data and complete data table/graph provided in Appendix 5
- Establish that $L_1^2 f_1 = L_2^2 f_2$
  - Note that the 10 cm pipe has 4 times the frequency as the 20 cm pipe.
- Based on the data they collected; ask participants if their hypothesis was correct. Participants will make calculations and figure out that by cutting the idiophone length in half, the frequency increases four times.
- To further explore this concept, relate the frequency of the pipes to defined musical notes.
  - Use Figure 15 to relate idiophone frequencies to a piano instrument
  - Re-measure the frequency of the 22 cm long pipe
  - Consult the table of note frequencies (Figure 15) to identify which note corresponds to the frequency generated by the pipe
    - For example, the tested 22.1 cm pipe yields a frequency of 1380 Hz which is close to the frequency of F₆ (1396 Hz Figure 15)
- With this information, calculate the length of a pipe that will produce the frequency of C₆ (1046 Hz), $L_{C6}$. (Figure 15.)
- Calculations:
  - $L_{F6}^2 f_{F6} = L_{C6}^2 f_{C6}$
  - $L_{C6}^2 = L_{F6}^2 \frac{f_{F6}}{f_{C6}}$
  - $L_{C6} = 22.1 \text{ cm} (\sqrt{1380}/1046)$
  - $L_{C6} = 25.4 \text{ cm}$
  - Note that these calculations are valid only for the sampled pipe.
Anticipated Time: 40

Pre-requisite/prior knowledge:

Formative Assessment:

- **Plans for Independent Student Practice**
  - Participants should visit the *Scientists for Tomorrow* website [www.scientistsfortomorrow.org](http://www.scientistsfortomorrow.org) as well as visit Pinterest, Vimeo and Facebook, so they understand what the program is about.

- **Closure** (Brief teacher or student-led review, with reference back to essential questions and enduring understandings)
  - Remind participants that through the application of scientific and mathematical principles we are able to calculate the length of the pipe that will produce a desired frequency, allowing us to design a musical instrument that will play the desired frequencies to provide the desired musical scale. Discuss the construction of a wind chime for the following class.
  - Review what was learned in class
  - Clean the room and put the tools and material back to its place
  - Anticipated Time: 15 minutes
LESSON OVERVIEW
Lesson 7 Subject and Topic: Designing a Chime
Grade Level(s): 6-8
Brief Description of Lesson: Participants will be introduced the concept of an 8-note musical scale and that each note has a specific frequency associate with it.

STAGE 1: IDENTIFY DESIRED RESULTS

- **Enduring Understanding(s):**
  - Calculating the length of idiophones to produce a desired frequency.

- **Essential question(s):**
  - What is the process of producing idiophones that generate specific frequencies?

- **Common Core State Standard(s)--Content:** N/A
- **CCSS Standard(s) for Mathematical Practice:** (Change to process standards for other subjects, as needed)

- **Key Content Knowledge and Skills**
  - Students will know:
    - The differences between low and high frequencies.
    - The importance of musical scales in producing music.
    - Instruments are categorized by the material that is vibrating to generate sound
    - The four types of instruments in an orchestra
    - How to design the chime
    - Determine the lengths of the pipes
  - Students will be able to:
    - Measure and determine the lengths of the pipes
    - Make a wind chime

- **Academic Language Knowledge and Skills**
  - Students will know:
  - Students will be able to:

- **English Language Development Knowledge and Skills (for starting, emerging, and developing ELLs)**
  - Students will know:
    - Vocabulary List: Frequency, Vibrations, Pitch, Tone and Node
  - Students will be able to
    - Use these terms in conversation with their peers

STAGE 2: DETERMINE ACCEPTABLE EVIDENCE

- **Pre-requisite/Prior knowledge for Both Content and Language**
  - When determining the frequency of an idiophone, we know that cutting a length in half produces four times the frequency.

- **Formative Assessment for Both Content and Language**
  - **Student Product and/or Performance**
    - Participants will build a wind chime
  - **Assessment Criteria**
    - Students will complete wind chimes with 95% accuracy

- **Summative Assessment:** N/A

STAGE 3: PLAN LEARNING EXPERIENCES

- **Time Required for Lesson Segments**
  - Set/Hook: 15 minutes
- Teacher Input: 30 minutes
- Guided Practice: 40 minutes
- Closure: 5 minutes

- **Grouping Arrangements**
  - Whole class
  - Small group(s)
  - Pairs
  - Cooperative groups
  - Individual

- **Materials and Technology**
  - Use the *Physics of Sound and Mathematics of Music Lesson Plan* Handbook as an instructional tool.
  - **Materials:** The instructor will need to collect materials from the Columbia College Chicago at 623 S. Wabash, room 600N.
    - 1.5 m of 1/2” diameter copper pipe for each group of 2 participants
    - Pipe cutters
    - Scratch Awl
    - Rulers
    - Markers
    - Calculators
    - Wood
    - Saws and mounts
    - 1 Hole punch
    - 1 Drill and mount
    - 1 Hammer
    - Eye hooks
    - Pipe cleaners
  - **Technology:** Instructors will need to get access from the site to use computers and a projector.
    - Computer with Microsoft Excel and “audio-tester” software and outfitted with a microphone.
    - A projector to show participants *Scientists for Tomorrow* website and handouts.

- **Teacher's Preparation**
  - Assemble materials, practice procedures beforehand, identify new vocabulary, organize workstations, etc.
  - The instructor will need to have computers and microphone with the audio tester
  - Set up the classroom by putting four chairs per table.
  - Set up the work stations
    - Drilling station
- Saw station
- Measuring Station
- Hook Station
  - As participants arrive, teacher should take attendance so they can fully complete instructor log afterwards.

- **Differentiated or Individualized Learning (i.e. non-reader, ELL-levels, gifted)**

- **Set/Hook**
  - Instructor will introduce the concept of an 8-note musical scale
    - Explain how each note has a specific frequency associated with it.
  - Anticipated Time: 15 minutes
  - Pre-requisite/prior knowledge: Careers in STEM
  - Formative Assessment:

- **DEMONSTRATION/LECTURE:**
  - Designing a Wind Chime:
  - Have the class work in pairs of two.
    - Every 2 participants will have a 150 cm of pipe to build 2 chimes
  - The class will need to determine how long the longest pipe should be in order to maximize the use of the 150 cm of pipe for the 2 chimes
    - Every pair of participants will need to generate 2 pipes each of lengths $L_0$, $L_4$, $L_7$, and $L_{12}$ and determine the lengths of the pipes
    - Note: All of the pipes for a single chime must be cut from the same piece of pipe or the sound will change.
  - The instructor will create an Excel Spreadsheet that relates 12 semitone numbers to the length using various starting lengths (for example start with a longest pipe of 22 cm and calculate whether there will be enough pipe to cut the three corresponding pipes from the 75 cm of pipe allotted for each participant) (Figure 16)

<table>
<thead>
<tr>
<th>Semitone #</th>
<th>Length (cm)</th>
<th>$20\sqrt{1/2^{(A24/12)}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20.0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>19.4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>18.9</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>18.3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>17.8</td>
<td>17.8</td>
</tr>
<tr>
<td>5</td>
<td>17.3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>16.8</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>16.3</td>
<td>16.3</td>
</tr>
<tr>
<td>8</td>
<td>15.9</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>15.4</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>15.0</td>
<td></td>
</tr>
</tbody>
</table>
Will determine that the optimal pipe lengths will be: L₀= 20 cm, L₄=17.8 cm, L₇=16.3 cm and L₁₂=14.1 cm
- Sum of pipes for one chime is 68.2 cm
- Assist participants in measuring and cutting pipes of defined lengths
- Recall that all pieces for 1 chime must be cut from the same piece of pipe
- Measuring and cutting must be done precisely as possible
- Design a method to hang pipes:
  - Ask participants how the pipes should be hung in order to make them sound the best
  - Have participants hold the pipes in different positions and test the sound of the pipes (Figure 17a)
  - Calculate the required position of the node: 0.23 multiplied by the pipe length
    - Node positions will be 4.6 cm for L₀, 4.1 cm for L₄, 3.8 cm for L₇, and 3.2 cm for L₁₂
  - Measure node position from the end of each pipe (Figure 17b)
  - Set up a “hole-drilling station” where all participants will be supervised in drilling holes to their pipes:
    - Using a Husky Metal Clamp, clamp the wooden mount to the table as seen in figure 17c. Then place the pipe in the wooden mount and clamp it with the Irwin Clamp
    - Use a hole punch and a hammer to initiate hole (Figure 17d)
    - Use drill to make a hole through the pipe at the node (Figure 17e)
Anticipated Time: 45 minutes

Formative Assessment:
- Observation of participation and appropriate tool use.

Plans for Guided Student Practice
- Build the Chime Support
  - Assist participants to cut a square piece of wood 9x9 cm (Figure 18a-18e)
  - The participants will draw 2 diagonals (corner to corner) on the piece of wood to make an X (Figure 18f-18h). Mark holes in the center and on each of the four corners (1.5 cm from the corner inwards) with a marker. Use the hammer and scratch awl to make indentations (figure 18i).
  - On the other side of the wood, place one hole in the center.
  - Place fish eye hooks into each hole (6 total) (figure 18j & 18k).
    - Note: If you are using open brass hooks, use the pliers to close the hooks
  - Put pipe cleaners through the holes of the copper pipes and attach it to the hooks on all four corners. On the opposite side, place a pipe cleaner through the center hook so the chime can hang.
  - Take a small piece of extra wood, and drill a small hole 2cm from the top edge. Cut a small piece of cooper pipe and attach a string to the piece of copper pipe by wrapping the string in and around the pipe. Then tie one end of the string to the extra piece of wood and the other end to the center hook of the pipe. (Make sure the wood hits the chimes and finally, decorate!)
- Anticipated Time: 40
- Pre-requisite/prior knowledge: Appropriate tool use
- Formative Assessment: Observation of appropriate tool use and knowledge of measurement
- **Plans for Independent Student Practice** - Participants should visit the *Scientists for Tomorrow* website www.scientistsfortomorrow.org, as well as to visit Pinterest, Vimeo and Facebook, so they understand that they are in an important program.

- **Closure** (Brief teacher or student-led review, with reference back to essential questions and enduring understandings)
  - Plans: Review of what was learned in the lesson
  - Anticipated Time: 5 minutes
APPENDIX 1 - Picture of an orchestra
APPENDIX 2 - Frequency of the Vibrating String and its Length

<table>
<thead>
<tr>
<th>Length (cm)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
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<tr>
<td>50</td>
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<td></td>
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<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>22.5</td>
<td></td>
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</tbody>
</table>

Frequency vs. Length of Vibrating string

![Graph showing frequency vs. length of vibrating string](image-url)
**APPENDIX 3 – Pythagorean Ratios**

<table>
<thead>
<tr>
<th>Note</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
<td>1/1</td>
<td>9/8</td>
<td>81/64</td>
<td>4/3</td>
<td>3/2</td>
<td>27/16</td>
<td>243/128</td>
<td>2/1</td>
</tr>
</tbody>
</table>

Length of the open string L1: ___________  Ln: L1* ratio

<table>
<thead>
<tr>
<th>Musical note</th>
<th>Ratio</th>
<th>Length of the string Ln</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1/1</td>
<td>L1</td>
</tr>
<tr>
<td>D</td>
<td>8/9</td>
<td>L1 * 8/9</td>
</tr>
<tr>
<td>E</td>
<td>64/81</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>3/4</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>2/3</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>16/27</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>128/243</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1/2</td>
<td></td>
</tr>
</tbody>
</table>
## APPENDIX 4 - Air column length and frequency

<table>
<thead>
<tr>
<th>Length (cm)</th>
<th>Frequency (Hz)</th>
</tr>
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<tbody>
<tr>
<td></td>
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</table>

Frequency vs. Length of the Air Column

![Graph showing frequency vs. length of the air column]
## APPENDIX 5 – Vibrating solid length and frequency

<table>
<thead>
<tr>
<th>Length (cm)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
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**Frequency vs. Length of the Vibrating Pipe**

![Graph showing frequency vs. length of the vibrating pipe.](image-url)